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Directivity of tsunami generated by subduction zone sources*

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Abstract. In a subduction zone, a tsunami source is usually situated above the continental bottom slope. Due to the wave refraction, the opposite slope of a deep water trench works like an optical lens for tsunami waves. The tsunami propagation process is studied using the wave-ray approximation. The tsunami amplitude reduction rate strongly depends on a relative (regarding a trench) location of the initial surface displacement. If the source boundary expands closer to a deep trench axis, then the initially circled front line of the leading tsunami wave will be formed into almost a flat shape after passing a trench. The same kind of tsunami behavior can be observed when a wave is generated by a submarine mudslide on a trench slope. A number of hypothetic tsunamis were simulated in the Alaska-Aleutian and in the Kuril–Kamchatka subduction zones. Some locations of possible tsunami sources are estimated as most dangerous for the Hawaiian population and infrastructure.

For the most part the tsunamigenic earthquakes occur at subduction zones. The presence of a deep water trench is the main characteristic feature of such a zone. The initial ocean surface displacement (a tsunami source) and earthquake epicenters are mainly located above the continental bottom slope. In Figure 1, the bottom relief [1] of the Kuril–Kamchatka region with historical tsunami source locations is presented [2]. The depth at the deep water trench axis is approximately twice as great as an average depth of the Pacific Ocean.

The process of tsunami generation and tsunami propagation was numerically studied in the 2D computational domain with a model bottom relief, where a depth value depends on only one spatial coordinate. This bottom topography is presented in Figure 2. On the left boundary, a depth is equal to 100 m. Then the depth value is linearly increasing up to 9000 m. In this case, there is a deep ocean trench axis. Farther to the right, the bottom surface is rising up to 3000 m and then becomes horizontal till the right boundary of the computational domain. The positive initial bottom surface displacement is located on the continental slope (see Figure 2).

The process of the tsunami wave refraction in an area of a variable depth was studied using the wave-ray approximation. The analytical solution for wave-ray traces above the sloping bottom [3] helps us to conclude that a greater part of the tsunami energy will be concentrated in the shoreward and

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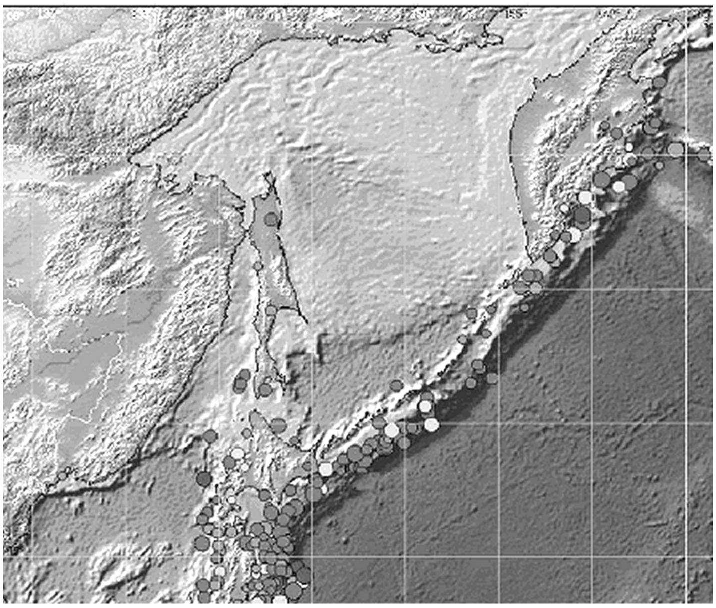


Figure 1. Location of some historical tsunami sources in the Kuril–Kamchatka subduction zone

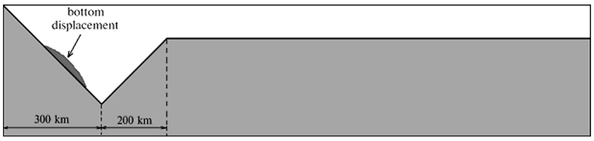


Figure 2. The model bottom relief and location of a tsunami source

in the seaward directions. In the area with a variable depth, the behavior of a tsunami wave ray is determined by the differential equations [4]:

$$\frac{d\vec{x}}{dt} = \frac{\vec{p}}{n^2(x)}, \qquad \frac{d\vec{p}}{dt} = \nabla \ln n(x), \tag{1}$$

where $\vec{x}$ is a spatial position of the wave-front point and $\vec{p}$ is the direction vector. Setting the unit vector of the initial ray outgoing direction from the source point x^0

$$\vec{x}\,\big|_{t=0} = x^0, \qquad \vec{p}\,\big|_{t=0} = n(x^0) \cdot \vec{\nu}^0, \tag{2}$$

and then solving numerically this differential problem (1)–(2), we can build wave-ray traces in the model computational domain. Wave rays of tsunami generated by a round-shaped source were determined using a simple numerical method and are shown in Figure 3 with white color.

The bottom relief of 1800 × 1200 grid-points computational domain is presented in Figure 3 (bottom). The results of wave-rays computation show

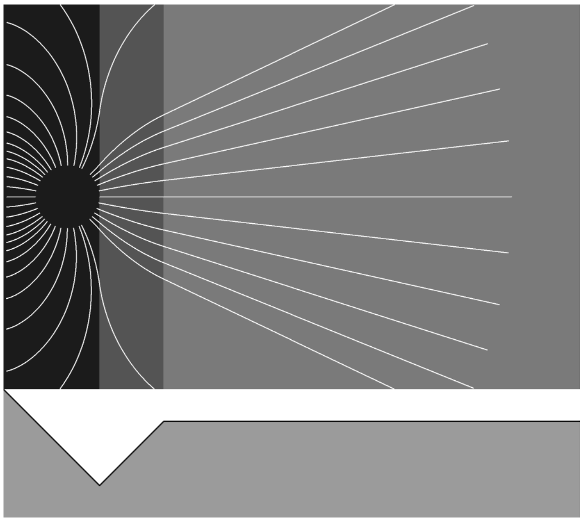


Figure 3. Wave-ray traces of tsunami generated by a round source

some ray focusing in the shoreward and in the seaward directions. The divergence of wave rays above the flat bottom is twice lower than their initial divergence. This means that the wave amplitude reduction in the right part of the computational domain will be twice less against the wave propagation without deep water trench.

This problem was also solved within a shallow-water model. Numerical modeling was realized using the MOST model [5]. The results of computational experiments were output as wave time series at selected grid-points and cumulated wave heights (a tsunami height maximum during the whole propagation process). The round-shaped initial water surface elevation (a tsunami source) is described by the formula

$$h(r) = \frac{h_0}{2}\left(1 + \cos\frac{\pi r}{R_0}\right), \quad 0 \le r \le R_0. \tag{3}$$

Here r is a distance to the source center, R_0 is the source radius, and h_0 is the vertical displacement value at the central point.

Figures 4 and 5 present a cumulative wave height in the whole computational domain for a 100 km round-shaped source with a deep trench (Figure 4) and in the area with a constant depth (Figure 5). The figure caption shows the correspondence of colors and wave height values in centimeters. The initial surface elevation at the central point was equal to 1 m. A more precise analysis of the wave radiation directivity can be made by comparison of the wave time-series at 20 grid-points located along the straight line orthogonal to the deep trench axis (see Figure 4). The first detector is located 200 km away from the source center and the last one is 2100 km away.

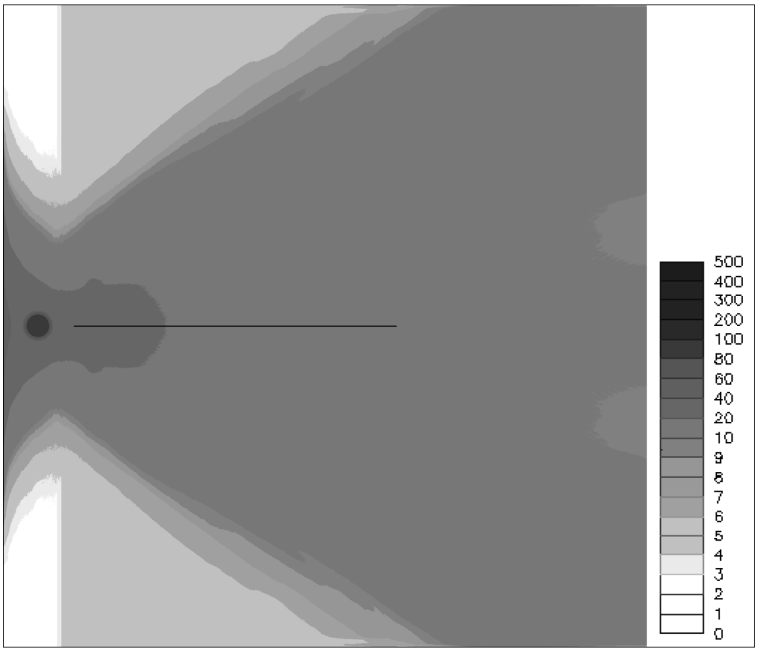


Figure 4. A cumulative wave height at all grid-points of 1800×1800 computational domain (a round-shaped source and a deep trench)

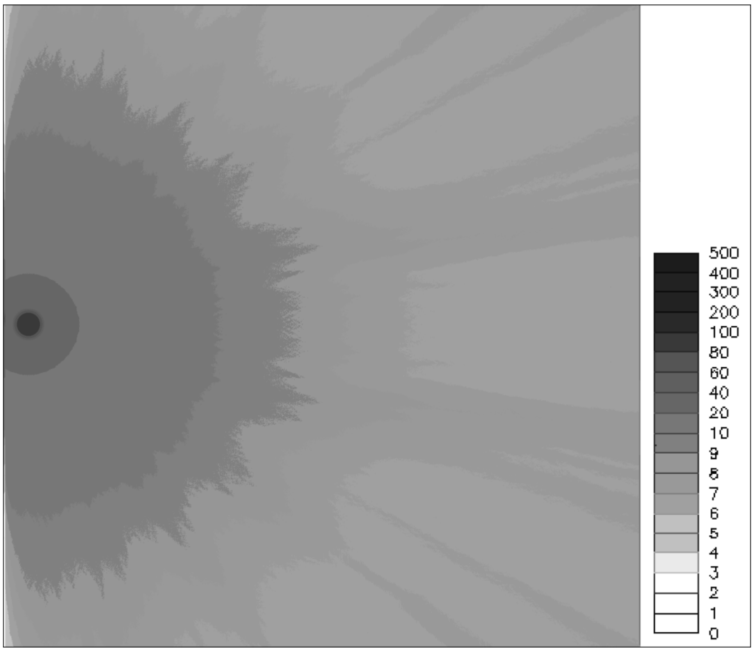


Figure 5. Distribution of wave height maxima at all grid-points of 1800×1800 computational domain (a round-shaped source and a constant depth)

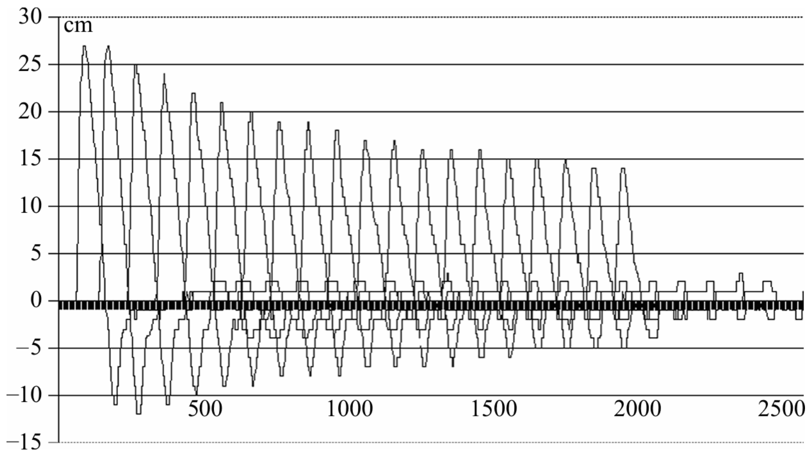


Figure 6. Tsunami time-series on the central horizontal line (a round-shaped source and a deep trench)

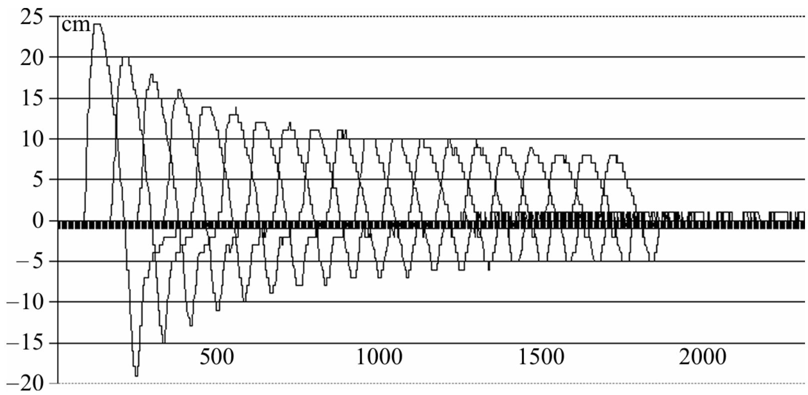


Figure 7. Tsunami time-series in the case of a constant depth (a round-shaped source)

Figure 6 presents the tsunami time-series from the round-shaped 100 km source in the case of a deep ocean trench. The vertical axis indicates to the wave amplitude and numbers along the horizontal axis mean time-steps. One can compare these results to the time-series of tsunami generated by the same source, but instead of a deep trench there was 3000 m deep flat bottom (Figure 7).

It is easy to see that at the grid-points, which are located far away from a source ($>$ 2000 km), the wave height is twice as great as in the case of a deep trench (15 cm against 8 cm in the case of a flat bottom). This result correlates well with estimations obtained by the wave-ray method (see Figure 3).

Figure 8

If instead of a round-shaped source we take an ellipsoidal tsunami source (Figure 8), then the directivity of the wave energy radiation will be more expressed. A prolate tsunami source radiates a greater part of the wave energy in its short axis direction (even if the bottom is flat).

Figure 9 presents the directivity of the wave energy radiation from 100 × 200 km ellipsoidal tsunami source in the case of a flat bottom. A comparison between the maximum wave height distribution for a flat (see Figure 9) and a deep trench (Figure 10) bottom relief guides to the same conclusions, as in the case of the round-shaped source. A deep ocean trench works like an optical lens and magnifies the tsunami wave amplitude in the orthogonal to the deep trench axis direction. The ratio between the wave amplitudes detected in these two cases (a deep trench and a flat bottom) also suggests the statement about the focusing effect of a deep ocean trench.

In the case of a uniform depth, the wave height is twice lower than in the case of a deep water trench (Figures 11, 12). The results presented in

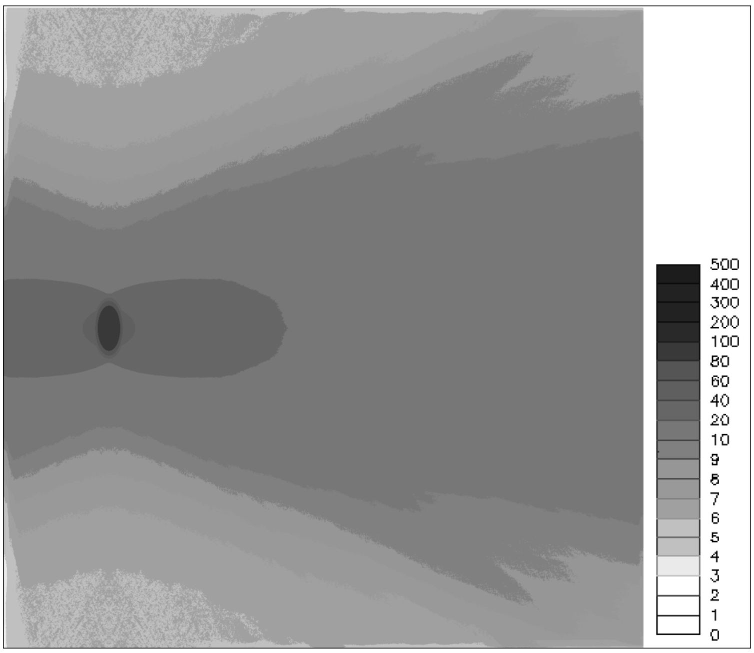


Figure 9. Maximum heights of the tsunami wave generated by an ellipsoidal source (a constant depth)

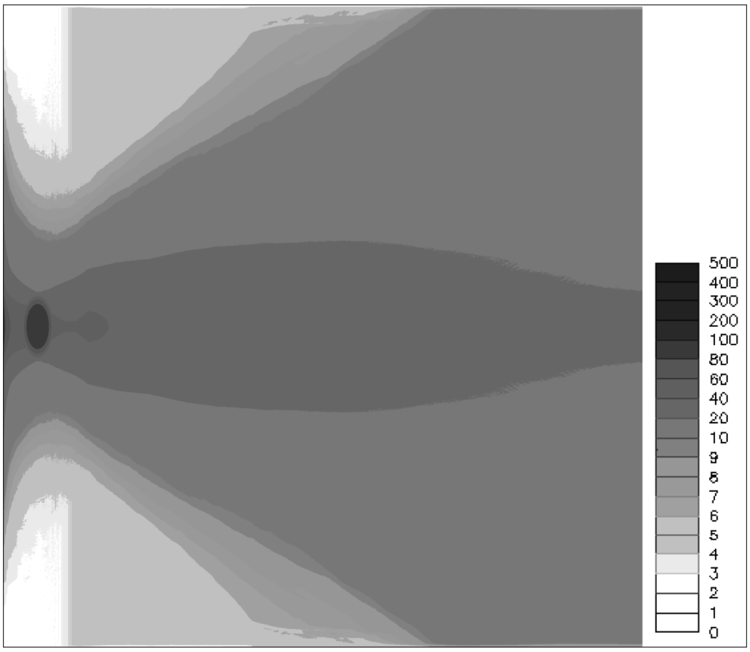


Figure 10. Maximum heights of tsunami wave generated by an ellipsoidal source (a deep trench)

Figures 11 and 12 show a low wave height decreasing rate in the case of a deep trench, as compared to the tsunami propagation above the flat bottom. In the most distant detector (2100 km away from the source center) the recorded wave height was equal to 27 cm against 17 cm in the flat case.

A highly expressed directivity of the tsunami wave radiation can sometimes occur in the Pacific. As an example, let us consider a 200×60 km ellipsoidal tsunami source located near the Aleutian Islands. Our objective is to find a potentially most hazardous position of such a source for Hawaiians. All the tsunami sources which were used in computations were of 200 cm high, but their position was varying. An example of such a source is presented in Figure 13.

As was earlier in the model computations, results are output as a cumulated wave heights chart (a maximum of the tsunami height during the whole propagating process). Such charts for two different source locations are presented in Figure 14.

As is seen from Figure 14a, the directivity of the highest tsunami waves from Source 1 cannot provide a hazardous wave on the densely-populated eastern Hawaii. These islands are not included into the sector of a maximum wave energy radiation. As far as Source 2 is concerned, the tsunami directivity from this source seems to be much more hazardous for the eastern

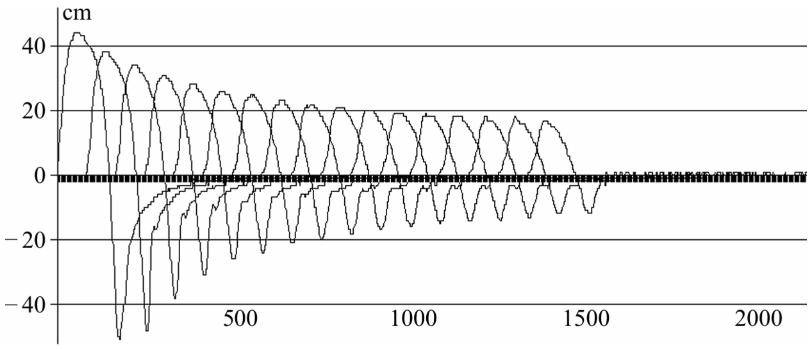


Figure 11. Tsunami time-series on the central horizontal line (an ellipsoidal source and a flat bottom)

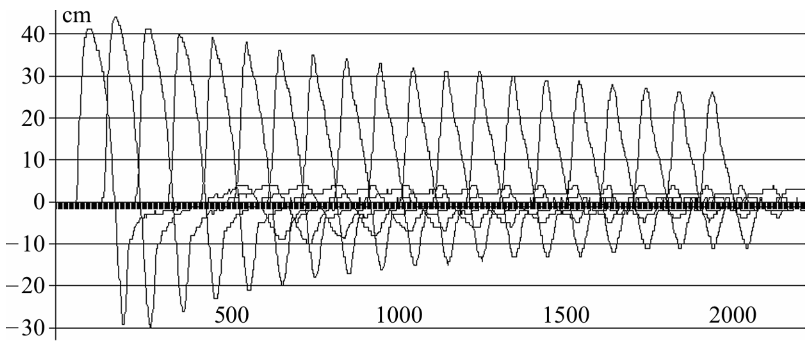
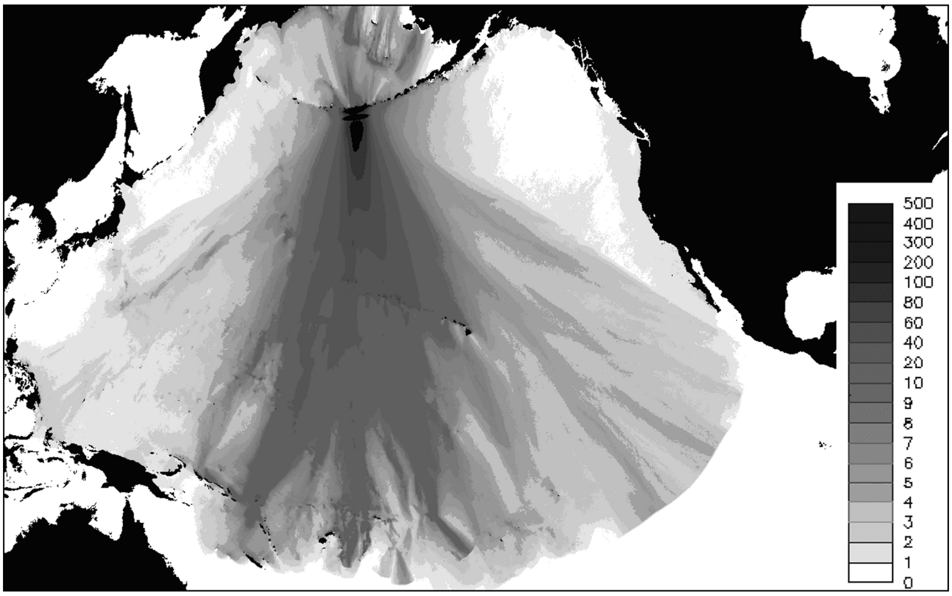
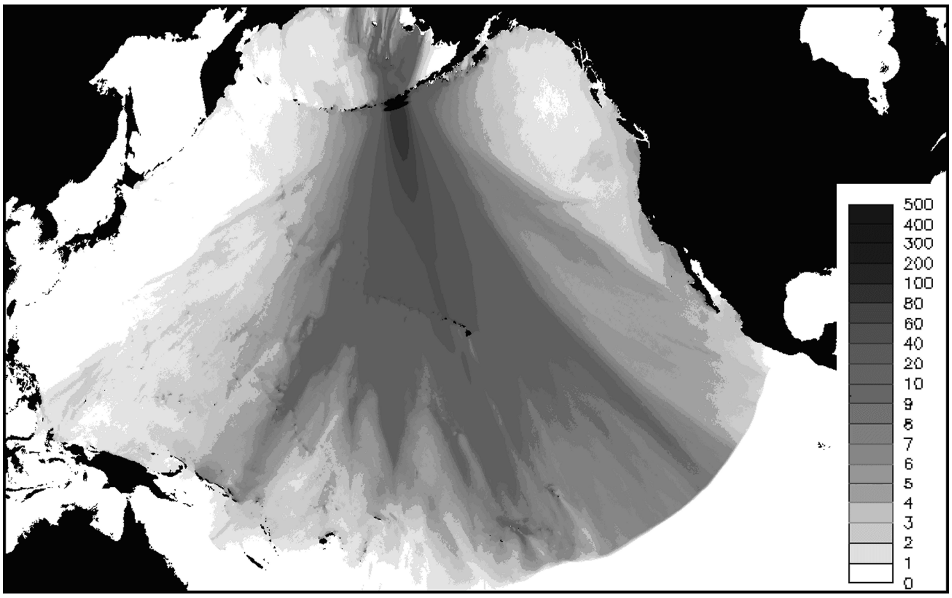


Figure 12. Tsunami time-series on the central horizontal line (an ellipsoidal source and a deep trench)

Figure 13. The Alaska-Aleutian subduction zone tsunami source location (200×60 km, initial elevation of 200 cm)



a



b

Figure 14. Distribution of tsunami height maxima from Sources 1 (a) and 2 (b)

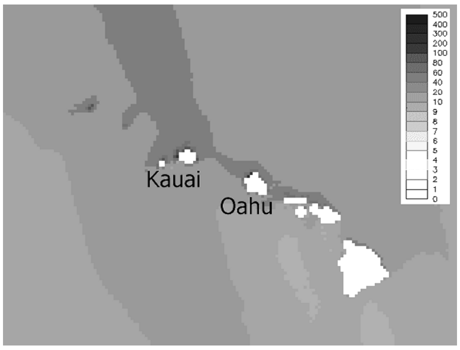


Figure 15. Maxima of tsunami wave heights around the Hawaiian islands (Source 2)

Hawaii, as compared to the first source (Figure 14b). The highest waves generated by Source 2 are directly moving to the Kauai and the Oahu islands. After zooming Figure 14b, one can see a detailed distribution of tsunami heights along the Hawaiian coast (Figure 15). It is seen (according to the color legend) that at the northern coast of these two islands the tsunami wave height exceeds 100 cm. Such an amplitude is observed just near the tsunami source.

Conclusions

The slope of a deep water trench works like an optical lens for tsunami waves. Due to the wave refraction above the bottom slope, a greater part of the tsunami energy will be concentrated in the shoreward and in the seaward directions.

If the source boundary expands closer to the deep trench axis, then the initially circled front line of the leading tsunami wave will be formed into almost a flat shape after passing a trench.

Some tsunami sources of the Alaska-Aleutian subduction zone can be potentially hazardous for Hawaii, due to a highly expressed directivity of the tsunami energy radiation.

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